

angle ϕ is used. In stress intensity factor handbooks, e.g. Tada *et al.*[4], it is often easy to misinterpret $K(\Phi)$ as that stress intensity factor for point S (and not point P_1), Fig. 1, which is the intersection point of OS defined by the parametric angle Φ and the elliptical crack front. To avoid this confusion Fabrikant's $K(\phi)$ of eqn (18) to calculate K for a point such as P_1 on the elliptical crack front is preferred since it can be unambiguously defined by the polar angle ϕ and the polar radius OP_1 .

Now returning to Fabrikant's paper in which he asserted that eqn (17) of Kassir and Sih is incorrect, it seems that he has got mixed up with the two angles Φ and ϕ and has wrongly interpreted Φ as ϕ in the K -equation (17). Equation (5) in his paper is therefore wrong and it corresponds to the incorrect definition of K in which

$$K = \lim_{\rho \rightarrow c(\phi)} \{\sigma_{zz} 2[c(\phi) - \rho]^{1/2}\}. \quad (19)$$

Had he realized the angle in Kassir and Sih's K -formula of eqn (17) is in fact the parametric and not the polar angle he would have easily derived the "correct" K -formula of eqn (18) in terms of the polar angle. Both $K(\Phi)$ and $K(\phi)$ are correct as shown in this discussion and they refer to the *same* point on the elliptical crack front. Consequently, Figs 2 and 3 in Fabrikant's paper which purport to show the discrepancy between the "incorrect" and "correct" K -formulae are meaningless and misleading.

There is nothing wrong with Kassir and Sih's formula of eqn (17) but care must be taken that Φ is a parametric angle and not the polar angle as is assumed in Fabrikant's paper.

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REFERENCES

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AUTHOR'S CLOSURE

It was strange to read a discussion being much longer than the original paper. All the main objections raised by Zhang and Mai were responded to in my closure related to the remarks by Kassir and Sih [1], and will not be repeated here. The reader is addressed to the above-mentioned closure. Here I present some specific notes related to the discussion by Zhang and Mai.

(1) The real confusion is not in my paper, but in the book of Kassir and Sih (and some other books which I do not name here taking into consideration present experience) where ϕ on each drawing is clearly indicated as the polar angle, while now they claim that the same parameter ϕ in their formulae stands for a parametric angle. I repeat once again that my paper was sent to both Kassir and Sih two years ago, and if the situation was clear to them at that time, they could have responded with an explanation but they did not.

(2) Zhang and Mai do not seem to realize that their "simple method" is nothing but an over-complicated repetition of mine. Indeed, both derivations start with eqn (1). Three manuscript pages of tedious transformations lead to their eqn (16), which is equivalent to my eqn (3). I have used only several lines of text and one intermediate expression to arrive at eqn (3). One can easily deduce that my eqn (3) will still be correct if one replaces $c(\phi)$ by *any* $C(\Phi)$ (where Φ is not necessarily the parametric angle), provided that an appropriate relationship between ϕ and Φ is established. The reader can observe graphically the degree of over-complication by comparing Figs 1 and 2 of Zhang and Mai with Fig. 1 of my paper.

(3) I have never claimed that my eqn (5) was correct. Quite opposite, I have used its incorrectness as an argument in order to derive a correct one. My Fig. 2 cannot possibly be neither meaningless nor misleading: it does not denounce anybody else's results and presents the numerical data related to my eqn (10), correctness of which nobody disputes. My Fig. 3 illustrates the error which any unsuspected user will make if he were to take the numerous graphical data, given in the book of Kassir and Sih, at their face value, namely, considering that the horizontal axis represents the polar angle, as is indicated on the adjacent drawing. Is such a warning meaningless? And who is misleading?

(4) Here is a quotation from Zhang and Mai: "To avoid confusion Fabrikant's... equation... is preferred since it can be unambiguously defined by the polar angle..." Thank you, gentlemen. This was precisely the purpose of my paper, and I hope that the vast majority of readers understands it.

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